Model Uncertainty

Reporting Error of Surface Water Models

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Proposed Calibration Goals

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* Water level within 10 cm
* Velocity within 10 cm/sec
* Temperature within 1°C
* Dissolved oxygen within 2 mg/L
* Nutrient and algal concentration within 10-25% of the range of the field data
* TSS within 10-25% of the range of the field data—suggestion added 10/22/2013

RMSE accentuates the biggest outliers
AME averages the big outliers

		AME, m	RMS error, m
2001	230	0.058	0.065

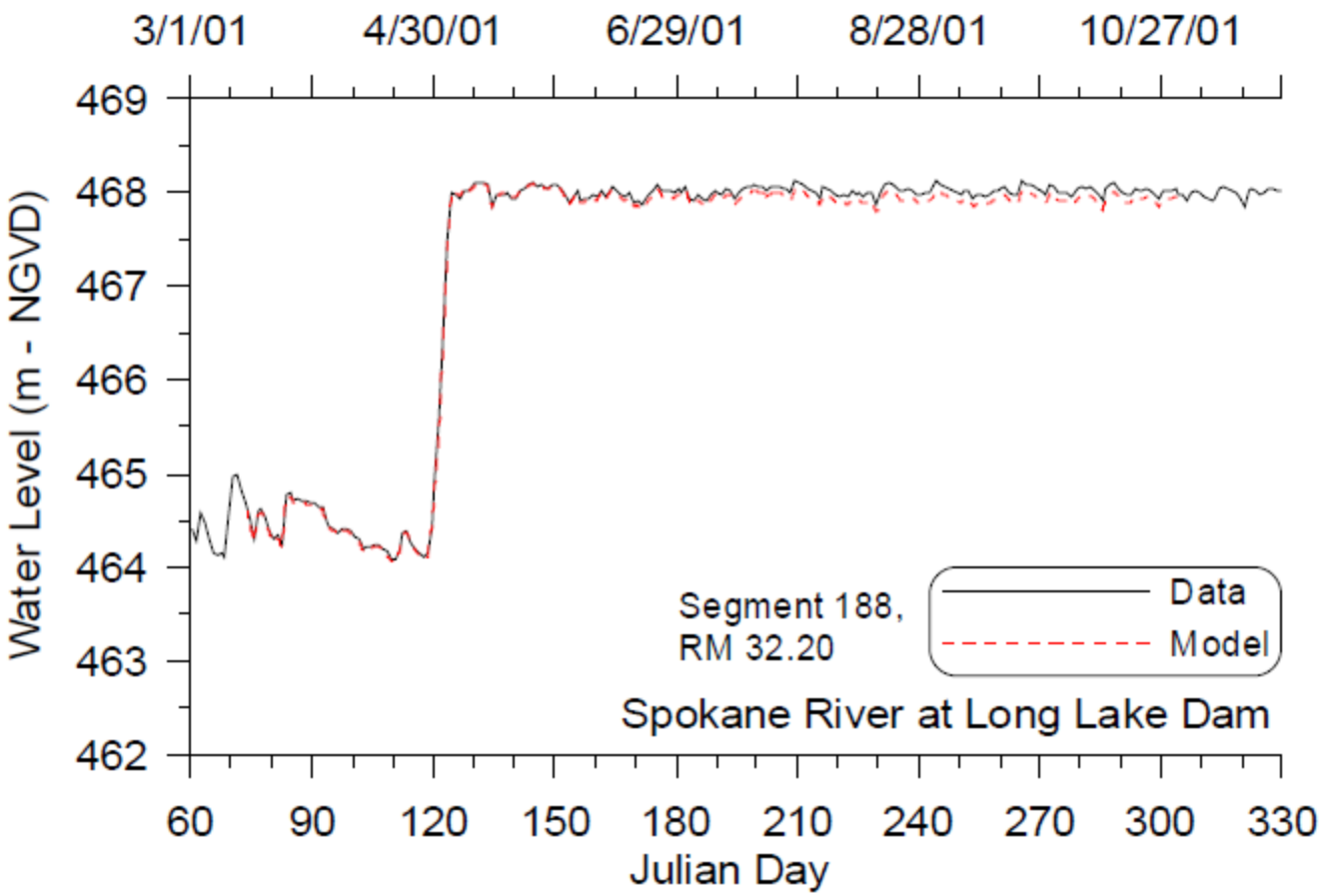


Figure 18. Water level prediction compared with data for the Spokane River at Long Lake Dam.

Image from 2001 calibration report for upper Spokane River model

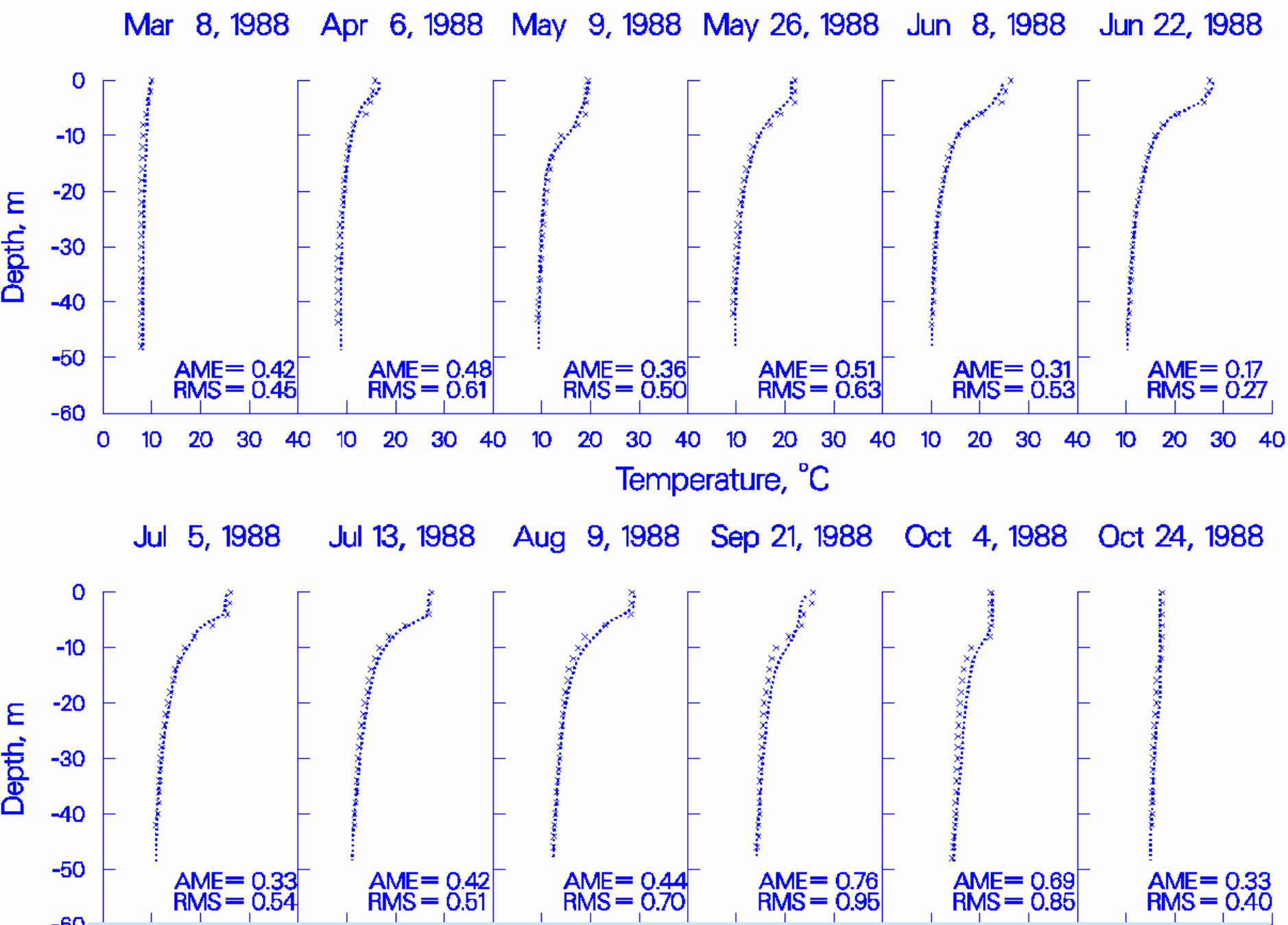


Image from www.ce.pdx.edu/w2/download _model_information.html

What is a model?

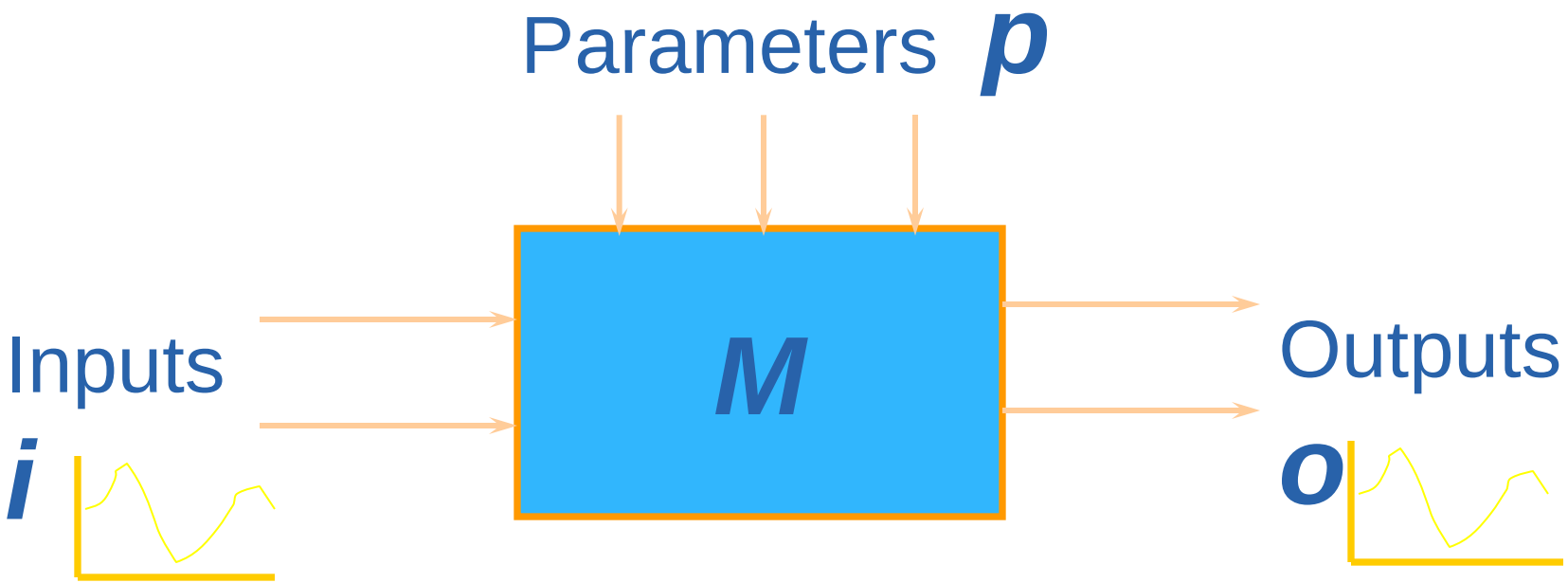
* A container of all expert knowledge
* A prediction of a future condition

The modeler's job is to quantify the model uncertainty

Three basic principles of modeling

1) The model is wrong.
2) I can never know exactly how wrong the model is.
3) I can only describe my propensity for error.

What a model does:-

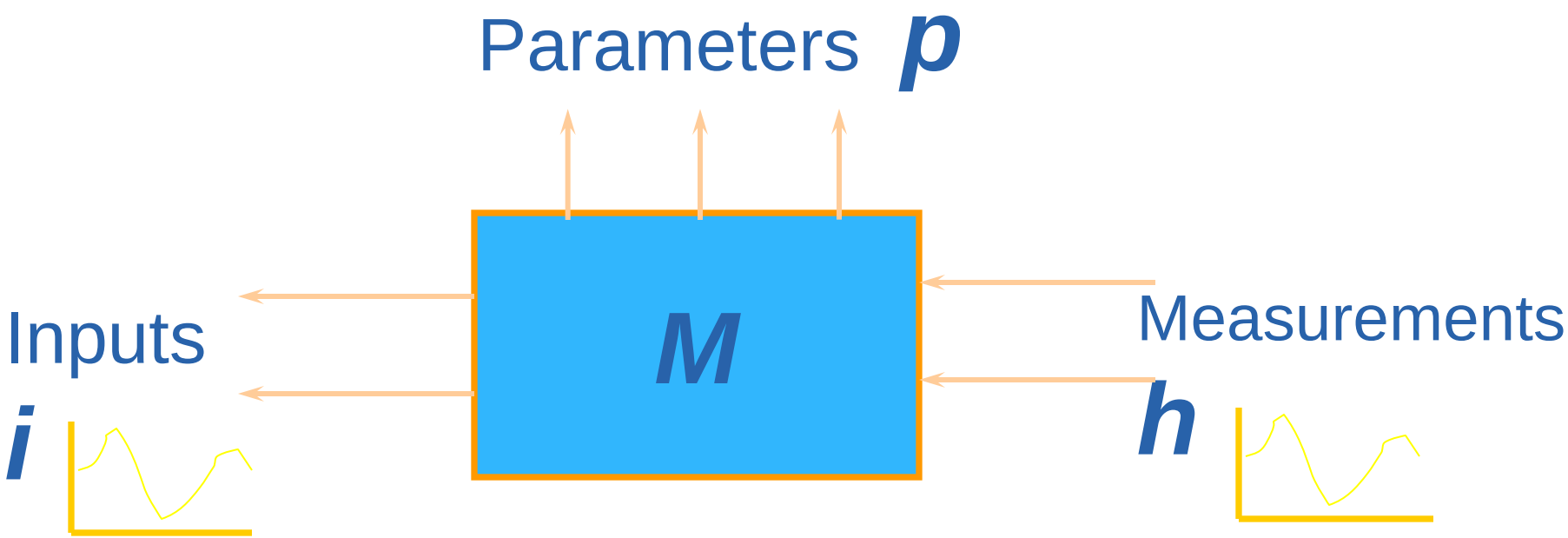


x describes system configuration

$$o = M(x,p,i)$$

Image from PEST workshop October 2013—Dr. John Doherty

The inverse problem:-

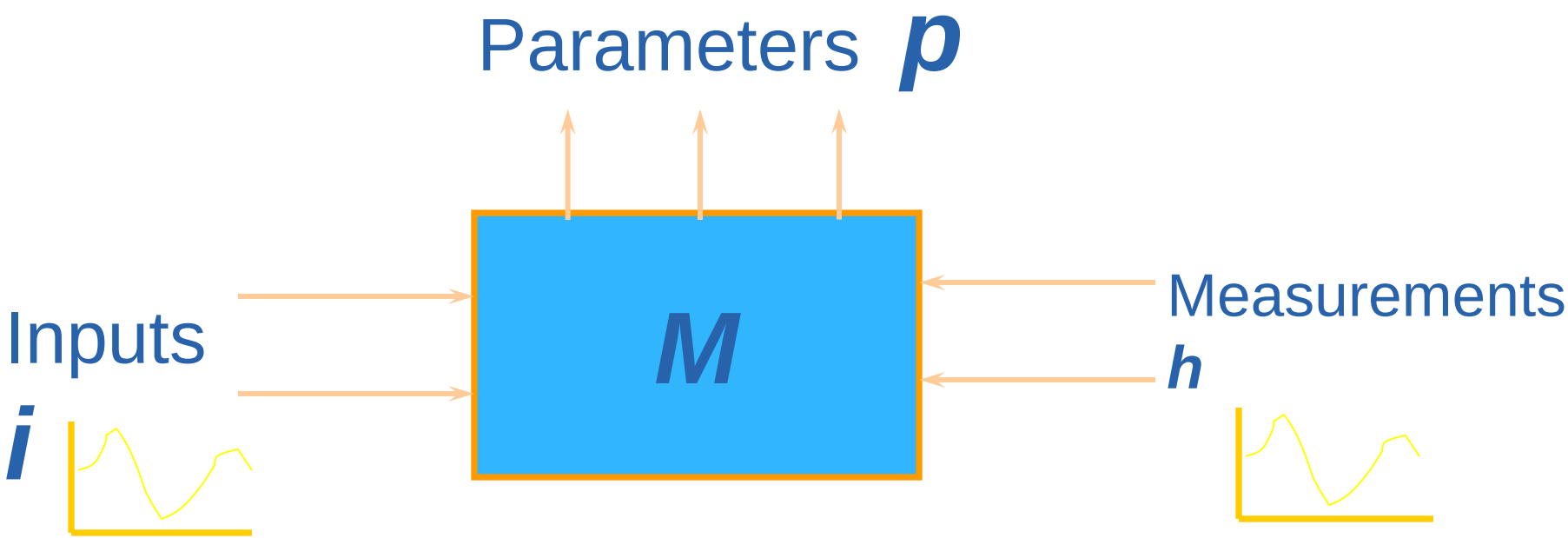


x describes system configuration

$$p, i = M^{-1}(x,h)$$

Image from PEST workshop October 2013—Dr. John Doherty

The inverse problem:-

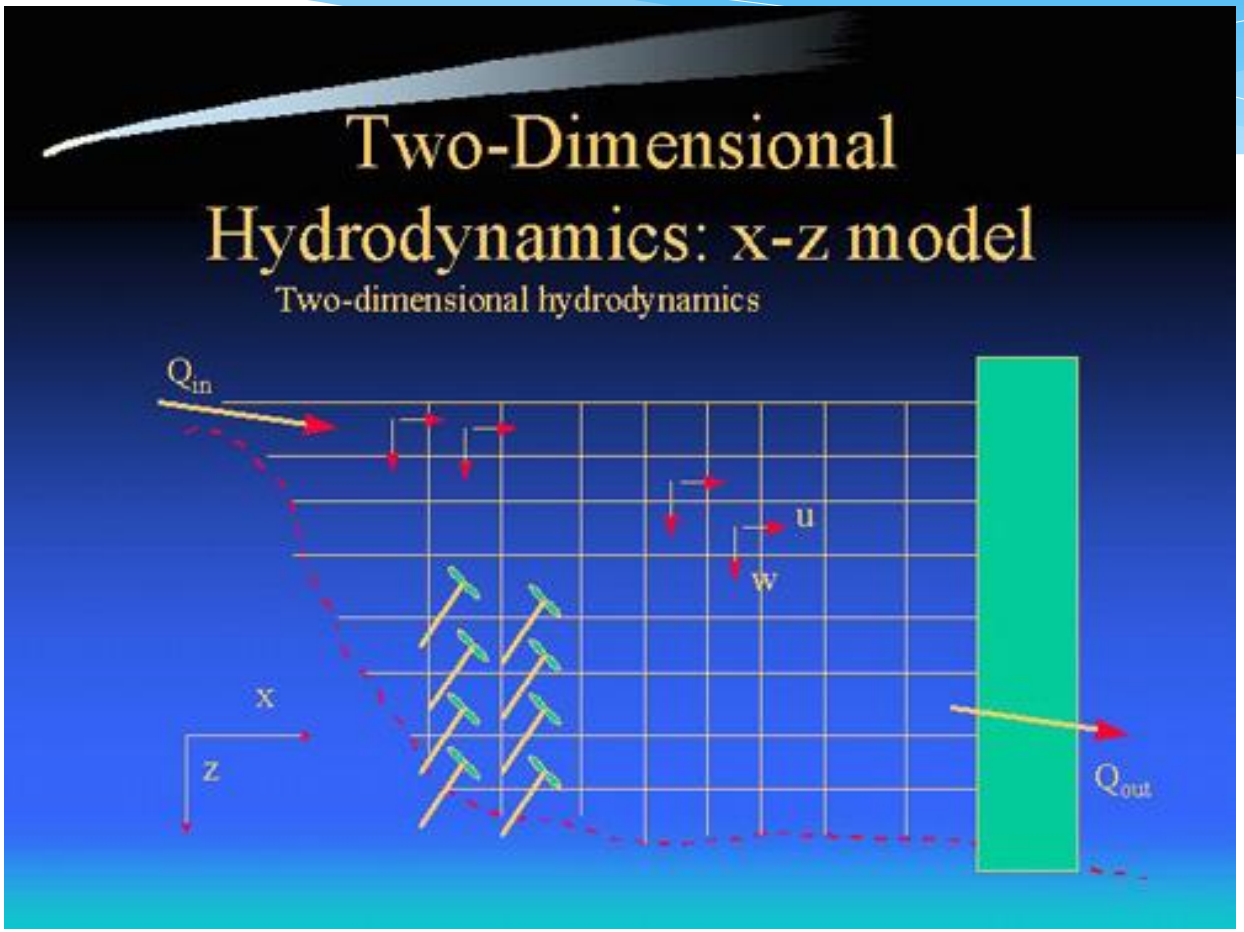


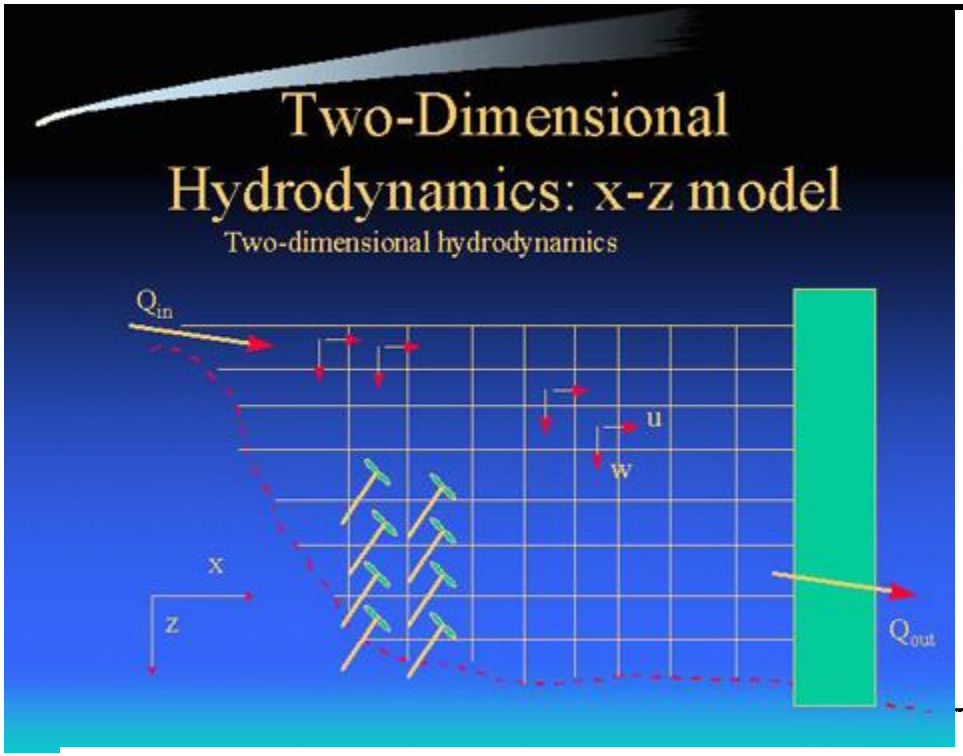
x describes system configuration

$$p = M^{-1}(x,i,h)$$

Image from PEST workshop October 2013—Dr. John Doherty

Solving the inverse problem works well in linear systems, but:



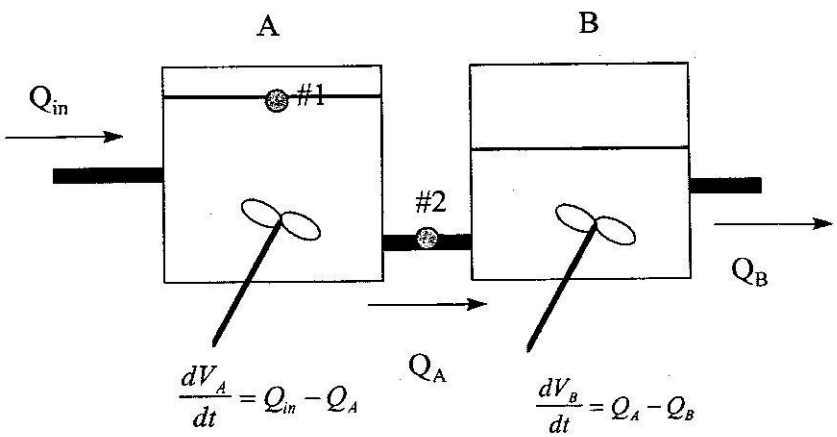


Continuity (or conservation of fluid mass)

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0$$

Conceptual Model

Fluid continuity $\frac{dV}{dt} = \sum Q_{in} - \sum Q_{out}$

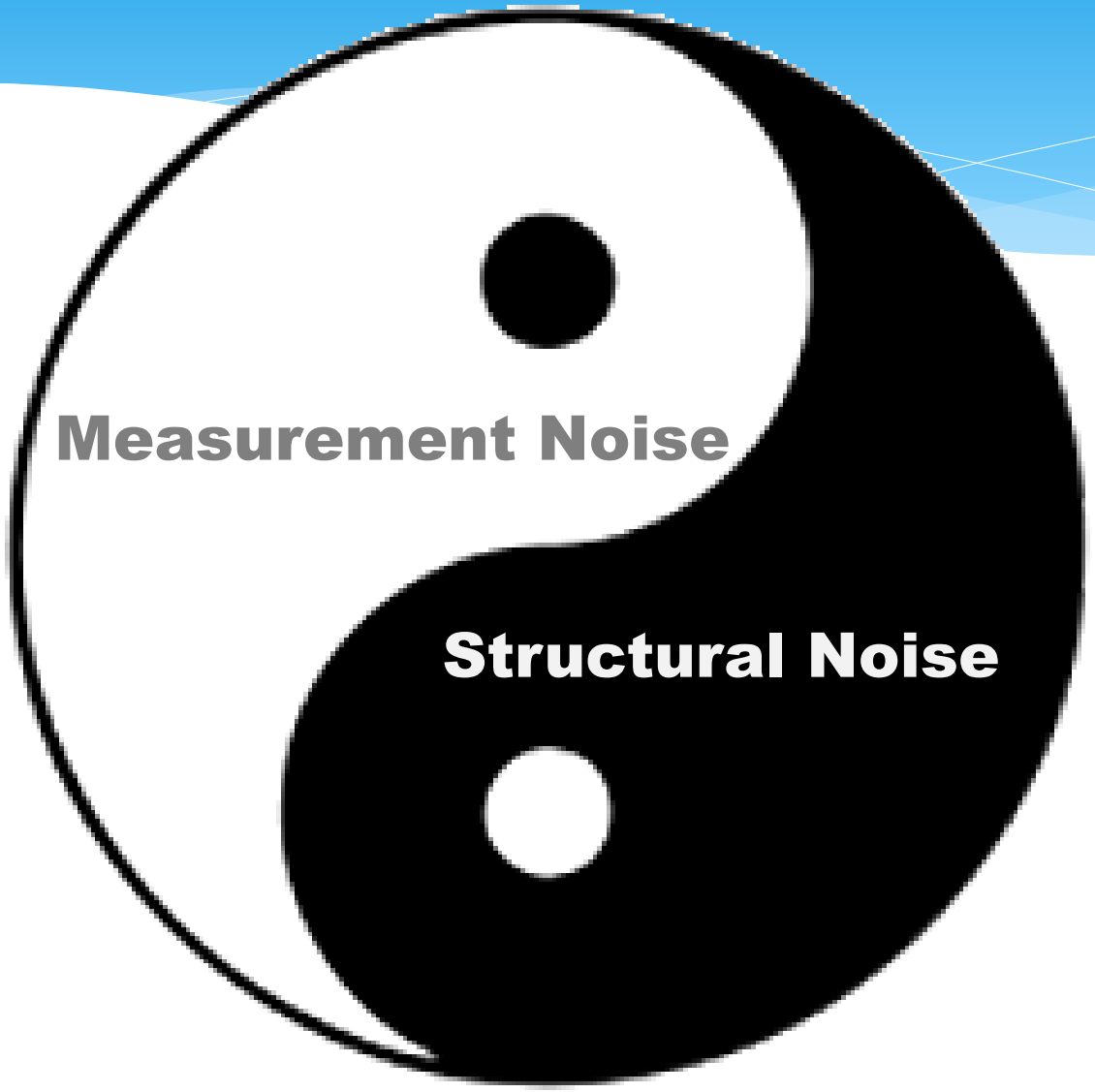


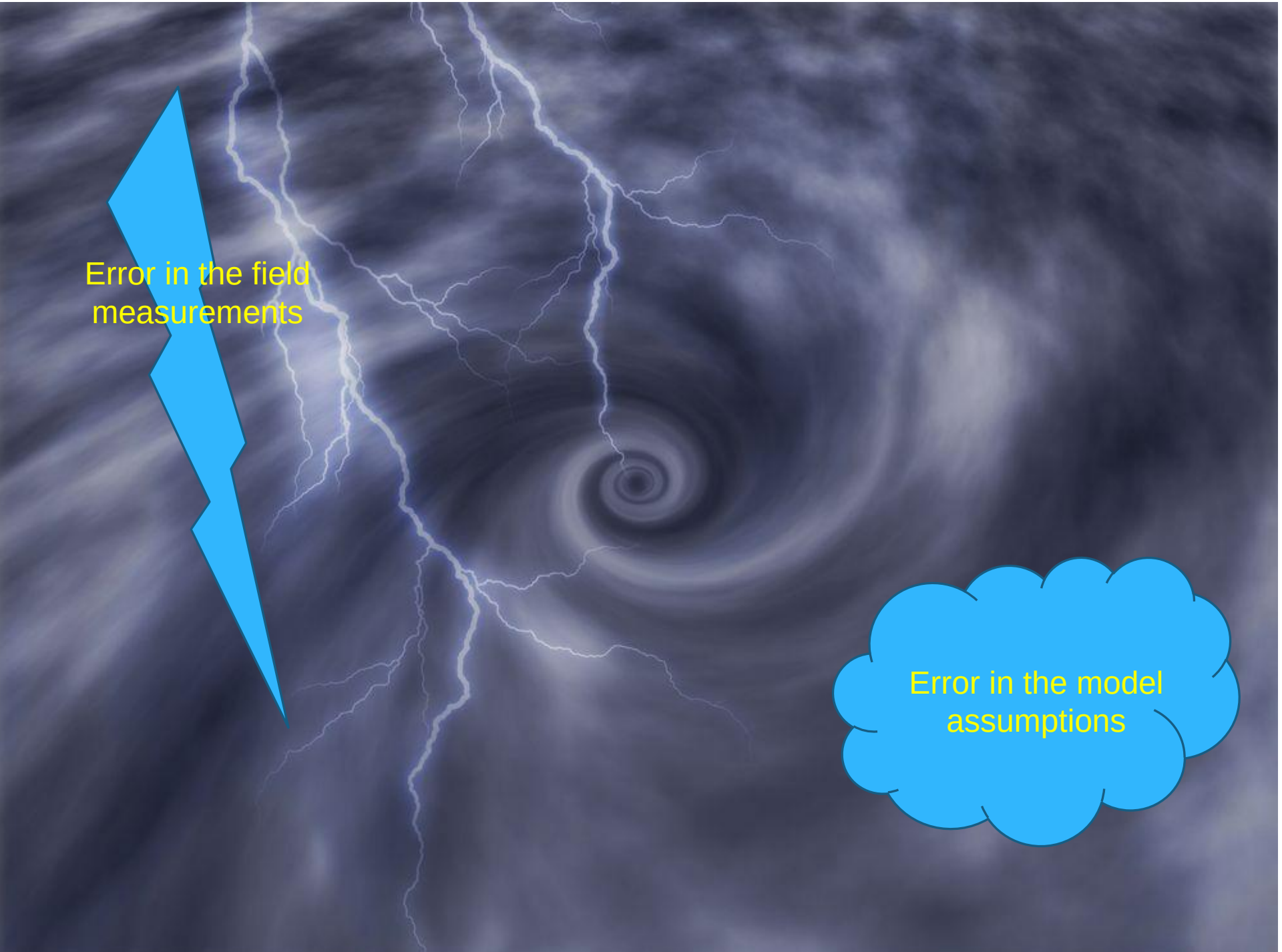
x-Momentum Equation

$$\underbrace{\frac{\partial \bar{u}}{\partial t}}_{1} + \underbrace{\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} + \bar{w}\frac{\partial \bar{u}}{\partial z}}_{2} \underbrace{- 2\Omega_z \bar{v} + 2\Omega_y \bar{w}}_{3} =$$

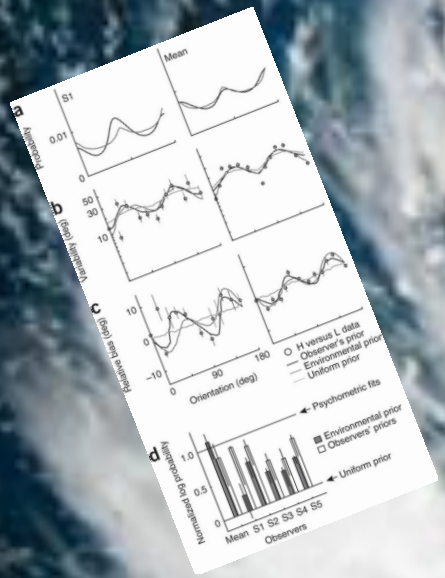
$$\underbrace{-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x}}_{4} + \underbrace{\frac{\mu}{\rho}\left(\frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\partial^2 \bar{u}}{\partial y^2} + \frac{\partial^2 \bar{u}}{\partial z^2}\right)}_{5} + \underbrace{\frac{1}{\rho}\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right)}_{6}$$

Sources of model error




Error in the field measurements
Error in the model assumptions

Perfect Storm



Surface Wave Equation

The meridional form of the surface ellipticalwave equation, equation (A-1), can be written in terms of the wave differences and its definitions given in Figure A-1. The vertically integrated horizontal wave equation can be written in finite difference over-averaged form as

$$F = \frac{-2\,BH_x^2}{g}\left[\frac{\partial p}{\partial x} - \frac{\partial(U^2HB)}{\partial x} - (UVB)_{y+}^{-} + (UVB)_{y-}^{+}\right] + \frac{\partial}{\partial x}\left[BH_x\frac{\partial U}{\partial x}\right] - (\tau_x B)_{y+}^{-} + (\tau_x B)_{y-}^{+} \qquad (A\text{-}18)$$

$$\left[u_i|v_i|\right]^2 = \left[u_i|v_o + at_i|\right]^2$$

$$= \left(u_i|v_o|\right)^2 + \left[u_i|a \times t_i|\right]^2$$

$$= \left(u_i|v_o|\right)^2 + \left(a \times t\right)^2\left[\left(\frac{u_i|a_i|}{a}\right)^2 + \left(\frac{u_i|t_i|}{t}\right)^2\right]$$

$$= \left(u_i|v_o|\right)^2 + t^2\left(u_i|a_i|\right)^2 + a^2\left(u_i|t_i|\right)^2$$

$$u_i|v_i| = \sqrt{\left(u_i|v_o|\right)^2 + t^2\left(u_i|a_i|\right)^2 + a^2\left(u_i|t_i|\right)^2}$$

definition of v

Rule 2

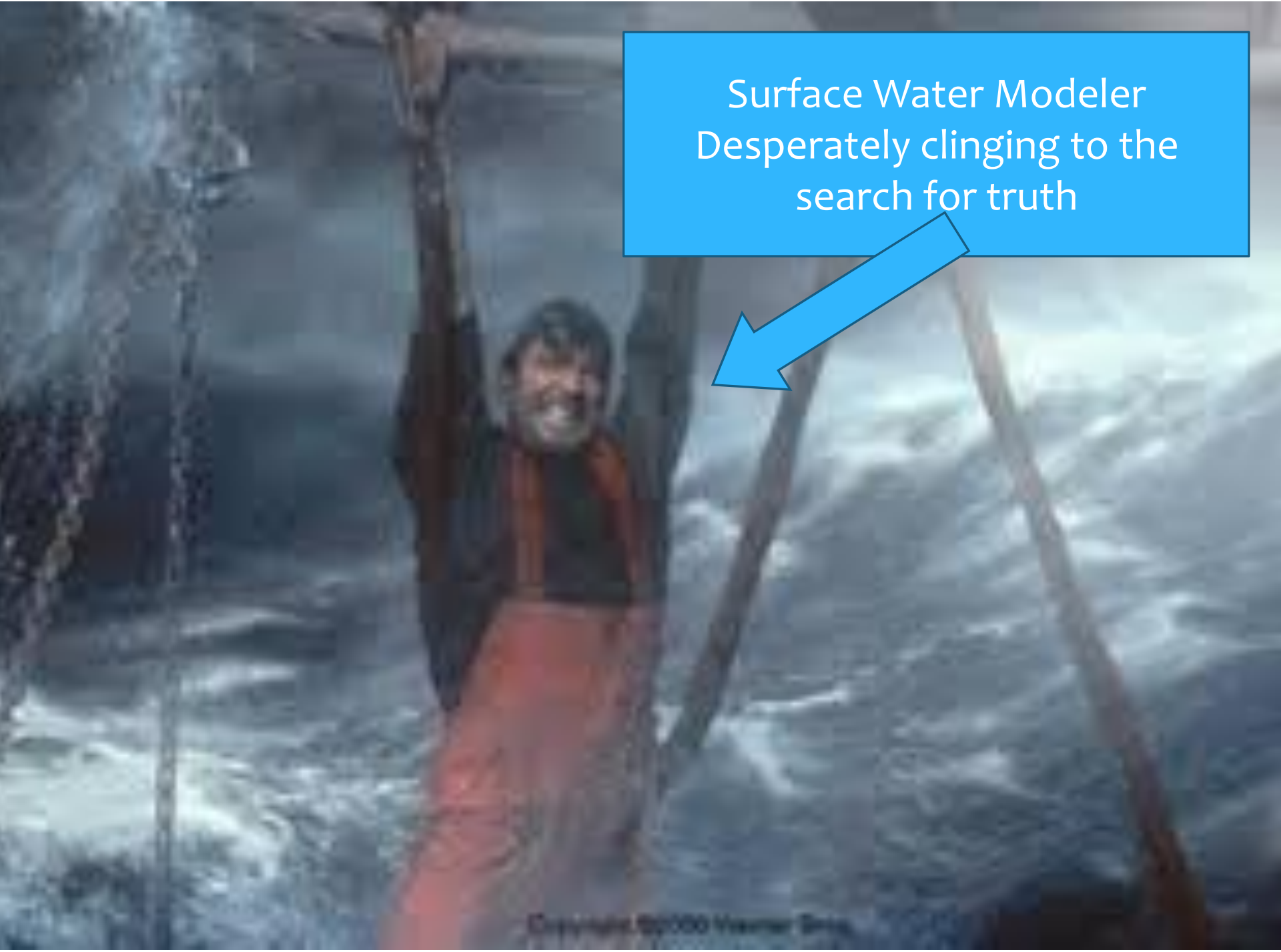
Rule 3

multiply through

square root of both sides

Horizontal Momentum

$$\frac{\partial UB}{\partial t} + \frac{\partial UUB}{\partial x} + \frac{\partial WUB}{\partial z} = -\frac{1}{\rho}\frac{\partial BP}{\partial x} + \frac{\partial\left[BA_x\frac{\partial U}{\partial x}\right]}{\partial x} + \frac{\partial Br_x}{\partial z} \qquad (A\text{-}1)$$


Surface Water Modeler
Desperately clinging to the
search for truth

Search
for Truth

Describing the propensity for error

Simple flow simulations in AQUATOX

$$\frac{dVolume}{dt} = Inflow - Discharge - Evap$$

where:

dVolume/dt	=	derivative for volume of water (m3/d),
Inflow	=	inflow of water into waterbody (m3/d),
Discharge	=	discharge of water from waterbody (m3/d), and
Evap	=	evaporation (m3/d), see **(3)**.

$$Velocity = \frac{AvgFlow}{XSecArea} \cdot \frac{1}{86400} \cdot 100$$

where

Velocity	=	velocity (cm/s),
AvgFlow	=	average flow over the reach (m^3/d),
XSecArea	=	cross sectional area (m^2),
86400	=	s/d, and
100	=	cm/m.

$$AvgFlow = \frac{Inflow + Discharge}{2}$$

where:

Inflow	=	flow into the reach (m^3/d);
Discharge	=	flow out of the reach (m^3/d).

Complex parameter sets in AQUATOX

AQUATOX- Edit Plant

Save | Cancel | OK | Print | GridMode | Export to Excel | Help

Plant Data:

Periphyton Linkage | Edit All

Plant: Peri Low-Nut Diatom | Scientific Name:

Search Names | Search Scientific Names

Plant Type: Periphyton | Toxicity Record: Diatoms | Edit All

Taxonomic Type: Diatoms

			References:
Saturating Light	64	Ly/d (Convert)	15WB; Hill 1996;Goldsborough & Robinson 1996
Use Adaptive Light			
Max. Saturating Light	300	Ly/d (Convert)	Default
Min. Saturating Light	80	Ly/d (Convert)	Default
P Half-saturation	0.006	mg / L	0.002, cal for WB, Borchardt, 1996 (0.006)
N Half-saturation	0.07	mg / L	0.008; Horne and Goldman, 1994, p. 140 = 0.07
Inorg. C Half-saturation	0.054	mg / L	" (greens)
Temp. Response Slope	2		DeNicola, 1996
Optimum Temperature	20	°C	DeNicola, 1996 (20-28)
Maximum Temperature	39	°C	DeNicola, 1996
Min Adaptation Temp.	2	°C	adapted to cold conditions
Max. Photosynthetic Rate	0.77	1 / d	calibrated; Fragilaria = 0.77
Photorespiration Coefficient	0.026	1 / d	same as high-nut
Resp Rate at 20 deg. C	0.08	g / g-d	Smith 1979, cited in C.& W.1983
Mortality Coefficient	0.001	g / g-d	prof. judgment
Exponential Mort. Coeff.	0.01	g / g-d	1%/d if phosynthesis = 0
P : Organics	0.018	ratio	Redfield et al., '63 stoichiometry
N : Organics	0.079	ratio	"
Light Extinction	0.03	1/m-g/m^3	0.14
Wet to Dry	5	ratio	<--- default
Fraction that is lipid	0.2	(wet wt.)	Default when loading simulation.

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AQUATOX- Edit Plant

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Resp Rate at 20 deg. C	0.08	g / g-d	Smith 1979, cited in C.& W.1983
Mortality Coefficient	0.001	g / g-d	prof. judgment
Exponential Mort. Coeff.	0.01	g / g-d	1%/d if phosynthesis = 0
P : Organics	0.018	ratio	Redfield et al., '63 stoichiometry
N : Organics	0.079	ratio	"
Light Extinction	0.03	1/m-g/m^3	0.14
Wet to Dry	5	ratio	<--- default
Fraction that is lipid	0.2	(wet wt.)	Default when loading simulation.

Phytoplankton Only:

Sedimentation Rate (KSed)	0	m / d	C & W '83
Temperature of Obs. KSed (estuary only)	0	°C	placeholder
Salinity of Obs. KSed	0	‰	placeholder
Exp. Sedimentation Coeff	0		if photosyn = 0, sed is 2X

Periphyton and Macrophytes Only:

Carrying Capacity (macrophytes)	80	g / m^2	ESF; Colby & McIntire '78 = 80
VelMax (macrophytes)	0	cm / s	N.A.
Reduction in Still Water (periphyton)	0.6	fraction	see VLimit.xls (Colby & McIntire '78 = 0.2)
Critical Force (FCrit for periphyton only)	0.005	newtons	reference, cal 0.001-->0.002
Percent Lost in Slough Event (periphyton)	90	percent	50% lost in sloughing event as default

If in Stream:

Percent in Riffle	50	%	
Percent in Pool	0	%	
Percent in Run	50.00	%	(All Biomass not in Riffle or Pool)

Salinity Effects (Estuary only):

Effect	Minimum Tolerance ‰	Maximum Tolerance ‰	Coeff1	Coeff2	Reference:
Photosynth.	0	0	0	0	placeholder
Mortality	0	0	0	0	placeholder

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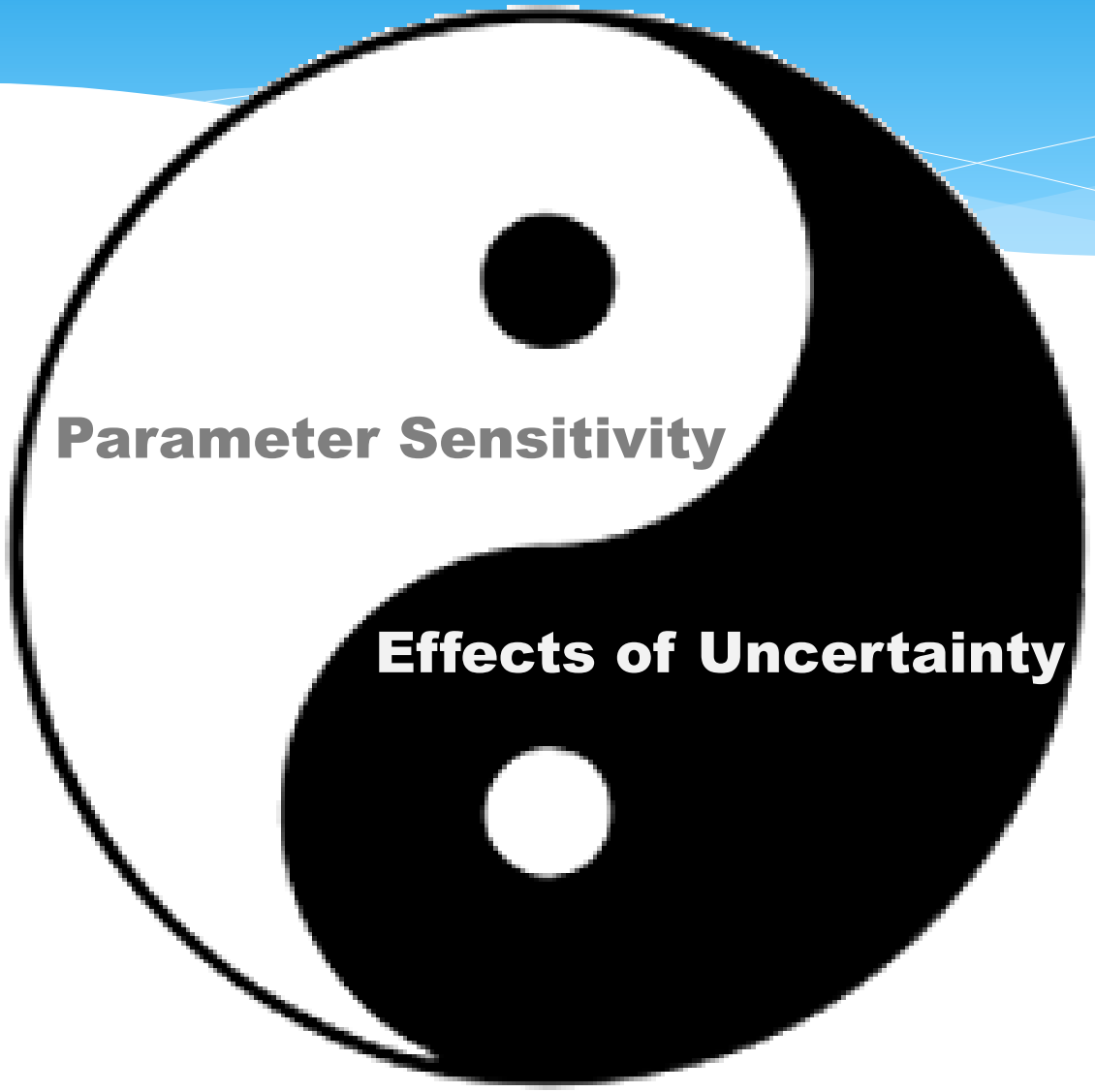
Reporting error



The scientific method

* Propose a hypothesis
* Test it against observations using an experiment
* Reject the hypothesis if it doesn’t fit data

Prediction Error

* The differences between values predicted by a model and the sample data are called residuals—we are reducing the residuals as much as possible during the calibration period.
* The difference between modeled values and actual values is called ***Prediction Error***

Field or laboratory measurements

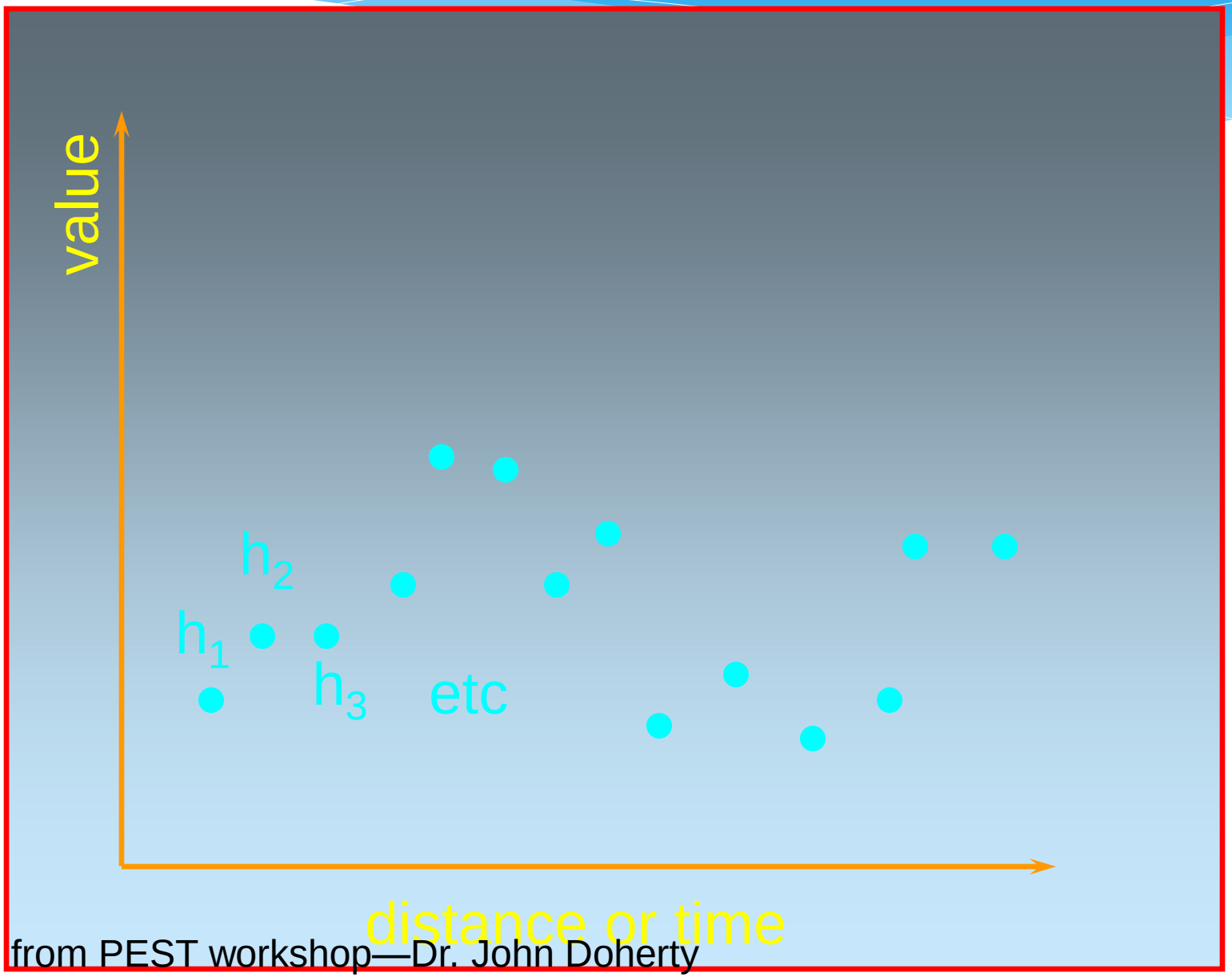
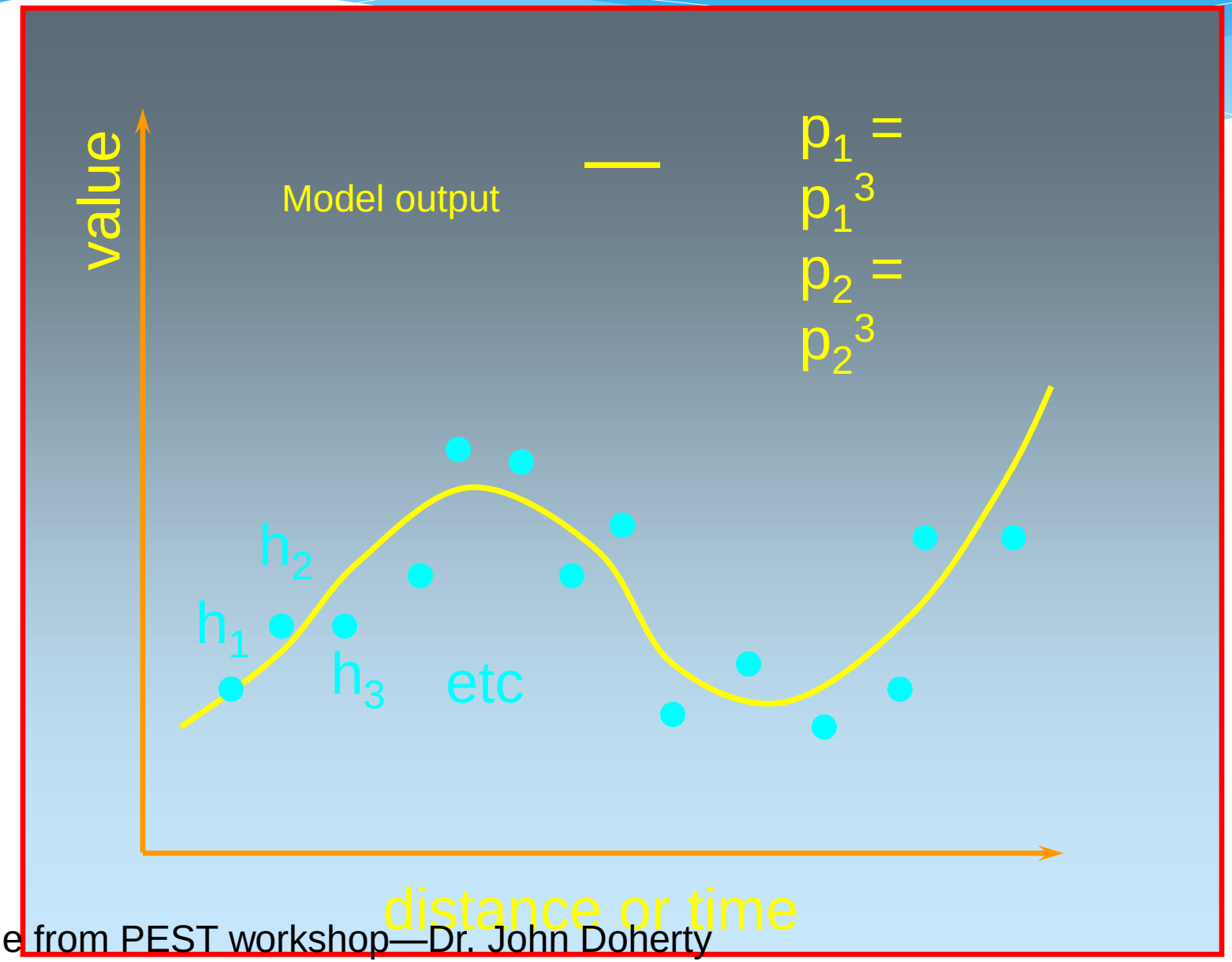


Image from PEST workshop—Dr. John Doherty

Field or laboratory measurements and model output



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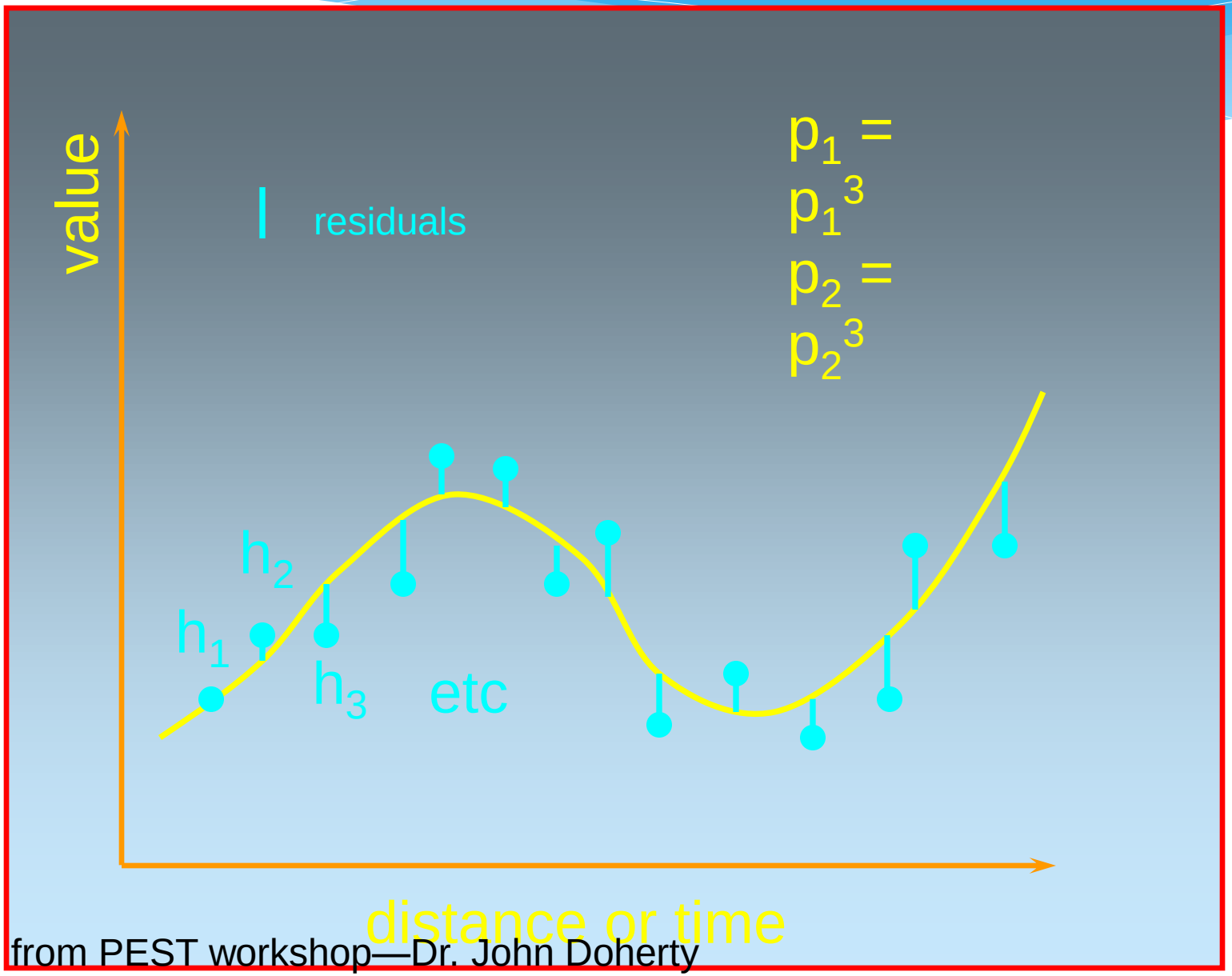
Residuals



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